Scattering of light exotic atoms in excited states

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The scattering of light exotic atoms in excited states ($n = 2 - 5$) from hydrogen has been calculated in a close-coupling model. For the first time, the absorption cross sections for hadronic atoms due to Stark collisions have been calculated by taking the shifts and widths of the nS states into account in a self–consistent quantum mechanical framework. A classical–trajectory Monte Carlo method has been used to calculate the scattering of exotic atoms from molecular hydrogen for $n \geq 8$. The Coulomb transitions with $\Delta n > 1$ are found to be the dominant de–excitation mechanism at the initial stage of the cascade.
Keywords: exotic atom, muonic hydrogen, pionic hydrogen, elastic scattering, Stark mixing, Coulomb de–excitation

1. Quantum mechanical close-coupling framework

The differential and total cross sections for the processes

$$(x^-p)_{nl} + \mathrm{H} \rightarrow (x^-p)_{nl'} + \mathrm{H}, \quad (x^- = \mu^-, \pi^-, K^-, \bar{p}) \tag{1}$$

have been calculated in a close–coupling model [1,2] by expanding the internal x^-p wavefunction into the set of n^2 eigenstates with principal quantum number n. The results have been obtained for $n = 2-5$ in a kinetic energy range relevant for atomic cascade calculations. Correct threshold behavior of the cross sections is ensured by including the energy shifts of the nS states due to vacuum polarization and strong interaction. In hadronic atoms, nuclear absorption is included in the model via the complex energy shift of the nS-states. The fully quantum mechanical close–coupling model allows us to calculate Stark mixing, elastic, and absorption cross sections in a self–consistent approach and to investigate the limitations of previously used semiclassical approximations [3,4].

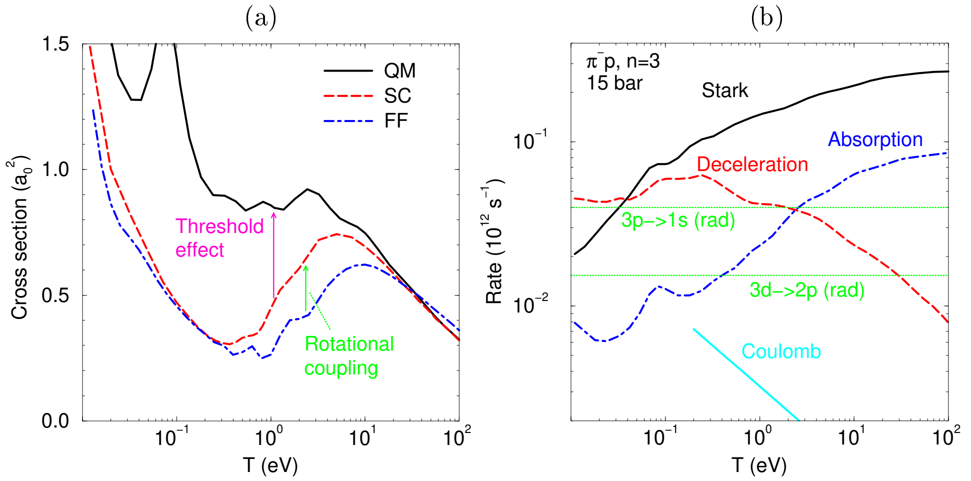

Figure 1. (a) The energy dependence of the l-averaged ($l > 0$) absorption cross sections for $\pi^- p$ scattering from hydrogen at $n = 3$. (b) The energy dependence of the l-averaged rates of Stark mixing, absorption, and deceleration for $(\pi^- p)_{n=3}$ at 15 bar in comparison with the radiative de-excitation rates $3p \to 1s$ and $3d \to 2p$ and the Coulomb de-excitation rate from [5].

The importance of correct threshold behavior and of using the correct angular coupling is clearly visible in the energy dependence of the nuclear absorption cross sections in hadronic atoms. Figure 1a shows the l-averaged absorption cross sections for $\pi^- p$ scattering from hydrogen at $n = 3$ obtained with different methods. The conventional semiclassical (eikonal) model with the correct angular coupling between the l–sublevels (SC) agrees with the quantum mechanical calculation (QM) only at $T > 10$ eV. The correct angular coupling increases the absorption cross sections for kinetic energies smaller than ~ 20 eV as can be seen from a comparison of the SC result with the fixed field model (FF) where the electric field from the hydrogen atom is assumed to be directed along the internal quantization axis of the exotic atom [3]. Because the semiclassical models do not take into account that different nl-channels have different momenta, they unavoidably fail in the near–threshold region.

A typical example of the l–averaged rates of Stark mixing, absorption, and deceleration for the $\pi^- p$ atom at $n = 3$ is shown in Fig. 1b in comparison with radiative, Auger, and Coulomb de–excitation rates. A few important observations can be made from the plotted energy dependences. The $\pi^- p$ atoms reaching the $n = 3$ state with energies larger than about 2 eV will most probably undergo nuclear absorption, while the less energetic ones ($T < 1$ eV) have a significant chance to be accelerated in Coulomb transition or, at sufficiently low density, to de–excite radiatively. For detailed studies of atomic cascades in $\mu^- p$ and $\pi^- p$ using the above described cross sections see [6] and references therein.

2. Classical–trajectory Monte Carlo calculation.

For large n, as a large number of quantum mechanical nl states gets involved in the collision, one expects that an adequate description can be obtained with classical methods. For this purpose, we have written a classical–trajectory Monte Carlo code to simulate the scattering of exotic atoms from hydrogen atoms or molecules. The following degrees of freedom are included in the model: the constituents of the exotic atom ($x^- = \mu^-, \pi^-, K^-, \bar{p}$ and the proton) and the hydrogen atoms are treated as classical particles, and the electrons are kept with fixed charge distributions corresponding to the $1S$ atomic state around the protons in the hydrogen atoms. In the case of molecular hydrogen, the $\mathrm{H} - \mathrm{H}$ interaction is given by the Morse potential. Given the initial kinetic energy and the quantum numbers n and l of the exotic atom, the cross sections are calculated as follows. A large set of impact parameters covering the interaction range is generated, and for each value of the impact parameter the classical equations of motions are solved numerically. As the initial state, a classical Kepler orbit for the exotic atom is constructed using the nl values and randomly oriented in space. The orientation of the hydrogen molecule is also taken to be random. The accuracy of the calculations is checked at regular intervals by controlling the energy and angular momentum conservation. When the exotic atom leaves the interaction region, the final Kepler orbit of the $x^- p$ system is projected onto the nl states. The scattering angle and, in case of molecular target, the energy absorbed by the hydrogen molecule are also obtained. The results are summarized in the form of differential and total cross sections.

Among many results of the Monte Carlo calculations, the most important one concerns molecular effects in the Coulomb de–excitation. While the transitions with $\Delta n = n_i - n_f = 1$ dominate for the atomic target (in agreement with ref. [5]), the transitions with $\Delta n > 1$ have been found to be strongly enhanced in the molecular case for $n_i > 10$ and the kinetic energy about 1 eV. The maximum of distribution over the final states n_f shifts towards larger values of Δn with the increase of n_i, so that the transitions with $\Delta n = 2 - 4$ are the most probable at the initial stage of atomic cascade in $\mu^- p$ and $\pi^- p$. The effect is demonstrated in Fig. 2a where the l–averaged Coulomb de–excitation cross sections for μp at

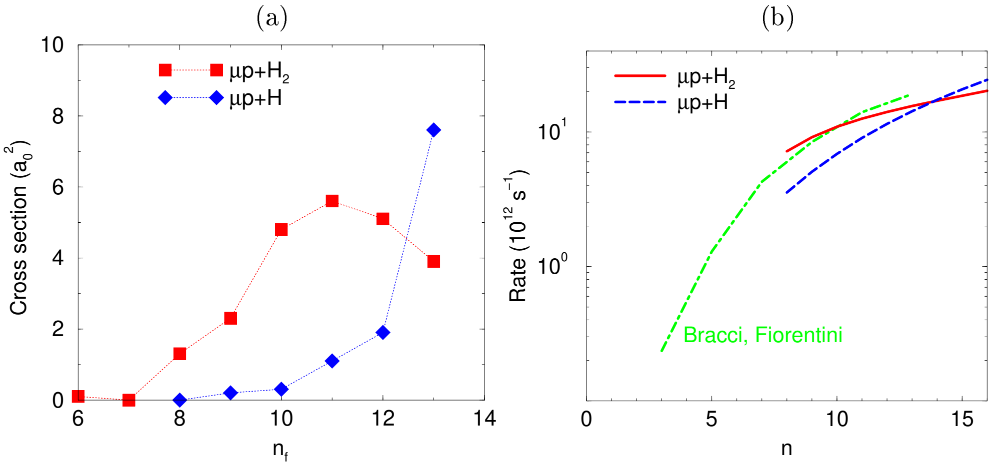

Figure 2. (a) The Coulomb de–excitation cross sections for $(\mu p)_{n_i=14}$ scattering from atomic and molecular hydrogen as a function of the final state n_f. (b) The n dependence of the total Coulomb de–excitation rates for atomic and molecular targets at liquid hydrogen density . The semiclassical estimate is according to Bracci and Fiorentini [5]. All plots correspond to the laboratory kinetic energy $T = 1$ eV.

$n_i = 14$ are shown as functions of the final state n_f. This has important implications for the atomic cascade as discussed in detail in [6]. Despite of significant differences in the distributions over the final states for molecular and atomic targets, the corresponding total Coulomb de–excitation rates are quite close to each other as shown in Fig. 2b. The classical–trajectory Monte Carlo results are also in a fair agreement with the semiclassical estimate from [5].

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